Derivatives of Log functions

① (a) $y = x^a$

$y' = ax^{a-1}$

(b) $y = a^x$

$y' = a^x \ln a$

(c) $y = x^x$

$\ln y = x \ln x$

$\frac{1}{y}\frac{dy}{dx} = (1)\ln x + \frac{x}{x}$

$\frac{dy}{dx} = (\ln x + 1)(x^x)$

(d) $y = a^a$

$y' = 0$

② $y = \log_3 \frac{x\sqrt{x-1}}{2} = \log_3 \frac{x}{2}(x-1)^{1/2}$

$= \log_3 \frac{x}{2} + \log_3 (x-1)^{1/2}$

$= \log_3 \frac{x}{2} + \frac{1}{2}\log_3 (x-1)$

$y' = \frac{1}{x/2} \cdot \frac{1}{\ln 3} \cdot \frac{1}{2} + \frac{1}{2} \frac{1}{x-1} \cdot \frac{1}{\ln 3}$

$y' = \frac{2}{x} \cdot \frac{1}{2\ln 3} + \frac{1}{2(x-1)\ln 3} = \frac{1}{x\ln 3} + \frac{1}{2(x-1)\ln 3}$

③ $y = x^{3/2} \log_2 \sqrt{x+1} = x^{3/2} \log_2 (x+1)^{1/2}$

$y' = \frac{3}{2} x^{1/2} \log_2 (x+1)^{1/2} + x^{3/2} \frac{1}{(x+1)^{1/2}} \cdot \frac{1}{\ln 2} \frac{1}{2}(x+1)^{-1/2}$

$y' = \frac{3x^{1/2} \log_2 (x+1)^{1/2}}{2} + \frac{x^{3/2}}{2\ln 2\,(x+1)}$

④ $y = e^{2\ln x} = e^{\ln x^2} = x^2$

$\frac{dy}{dx} = 2x$

⑤ $x^2 - 3\ln y + y^2 = 10$

$2x - \frac{3}{y}\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\frac{\left(-\frac{3}{y} + 2y\right)\frac{dy}{dx}}{-\frac{3}{y} + 2y} = \frac{-2x}{-\frac{3}{y} + 2y}$

$\frac{dy}{dx} = \frac{-2x}{\left(-\frac{3}{y} + 2y\right)y} \cdot y$

$\frac{dy}{dx} = \frac{-2xy}{-3 + 2y^2}$

⑥ $\ln xy + 5x = 30$

$\frac{1}{xy} \cdot \left(1y + x\frac{dy}{dx}\right) + 5 = 0$
$-5 \quad -5$

$\cancel{xy}\left(\frac{y + x\frac{dy}{dx}}{\cancel{xy}}\right) = (-5)xy$

$y + x\frac{dy}{dx} = -5xy - y$
$-y$

$\frac{\cancel{x}\frac{dy}{dx}}{\cancel{x}} = \frac{-5xy - y}{x}$

$\frac{dy}{dx} = \frac{-5xy - y}{x}$

⑦ $x + y - 1 = \ln(x^2 + y\sqrt{2})$

$1 + \frac{dy}{dx} = \frac{1}{x^2 + y\sqrt{2}} \cdot \left(2x + \sqrt{2}\frac{dy}{dx}\right)$

$1 + \frac{dy}{dx} = \frac{1}{1 + 0} \cdot \left(2 + \sqrt{2}\frac{dy}{dx}\right)$

$1 + \frac{dy}{dx} = 2 + \sqrt{2}\frac{dy}{dx}$

$\frac{(1 - \sqrt{2})\frac{dy}{dx}}{1 - \sqrt{2}} = \frac{1}{1 - \sqrt{2}}$

$y - 0 = \frac{1}{1 - \sqrt{2}}(x - 1)$

8) $y = e^{-x}$

$y' = -e^{-x}$

$-\frac{1}{2} = -\frac{1}{e^x}$

$\frac{-e^x}{-1} = \frac{-2}{-1}$

$e^x = 2$

$\ln e^x = \ln 2$

$x = \ln 2$

$2x - y = 8 \rightarrow m = 2$

$\perp m = -\frac{1}{2}$

$y = e^{-x}$

$y = e^{-\ln 2} = e^{\ln \frac{1}{2}} = \frac{1}{2}$

$(\ln 2, \frac{1}{2})$

9) $y = \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x-5)^3}} = \left(\frac{(x-3)^4(x^2+1)}{(2x-5)^3}\right)^{1/5}$

$\ln y = \frac{1}{5}\ln(x-3)^4(x^2+1) - \frac{1}{5}\ln(2x-5)^3$

$\ln y = \frac{4}{5}\ln(x-3) + \frac{1}{5}\ln(x^2+1) - \frac{3}{5}\ln(2x-5)$

$\frac{1}{y}\frac{dy}{dx} = \frac{4}{5(x-3)} + \frac{2x}{5(x^2+1)} - \frac{3 \cdot 2}{5(2x-5)}$

$\frac{dy}{dx} = \left(\frac{4}{5(x-3)} + \frac{2x}{5(x^2+1)} - \frac{6}{5(2x-5)}\right) y$

$\frac{dy}{dx} = \left(\frac{4}{5(x-3)} + \frac{2x}{5(x^2+1)} - \frac{6}{5(2x-5)}\right)\left(\sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x-5)^3}}\right)$

(10) $y = x^{\frac{1}{\ln x}}$

$\ln y = \ln x^{\frac{1}{\ln x}}$

$\ln y = \frac{1}{\ln x} \ln x$

$\ln y = 1$

$y \cdot \frac{1}{y}\frac{dy}{dx} = 0 \cdot y$

$\frac{dy}{dx} = 0$