

Worksheet: Improper Integrals

1. Evaluate the following integrals.

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| (1) $\int_0^1 \frac{1}{x} dx$ | (2) $\int_1^{\infty} \frac{1}{x} dx$ | (3) $\int_0^{\infty} xe^{-x} dx$ |
| (4) $\int_0^{\infty} \frac{1}{1+x^2} dx$ | (5) $\int_5^{\infty} \frac{1}{\sqrt{x-1}} dx$ | (6) $\int_0^1 \frac{1}{1-x} dx$ |
| (7) $\int_1^{\infty} \ln x dx$ | (8) $\int_0^1 \frac{1}{\sqrt{x}} dx$ | (9) $\int_0^1 \frac{1}{\sqrt{1-x}} dx$ |
| (10) $\int_0^{\infty} e^{-x} dx$ | (11) $\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}} dx$ | (12) $\int_{-\infty}^1 e^x dx$ |
| (13) $\int_{-\infty}^0 x^2 e^x dx$ | (14) $\int_{-\infty}^{\infty} xe^{-x^2} dx$ | (15) $\int_e^{\infty} \frac{1}{x(\ln x)^2} dx$ |
| (16) $\int_{-\infty}^{\infty} x^5 dx$ | (17) $\int_{-\infty}^{\infty} \frac{1}{x^2+16} dx$ | (18) $\int_1^{\infty} \ln x dx$ |
| (19) $\int_0^{\infty} e^{-2x} dx$ | (20) $\int_3^{\infty} \frac{1}{x^3} dx$ | (21) $\int_0^4 \frac{1}{(4-x)^{\frac{3}{2}}} dx$ |
| (22) $\int_0^2 \frac{1}{\sqrt{4-x^2}} dx$ | (23) $\int_0^4 \frac{x}{\sqrt{16-x^2}} dx$ | (24) $\int_0^{\frac{\pi}{2}} \tan \theta d\theta$ |
| (25) $\int_0^{\frac{\pi}{2}} \frac{1}{1-\sin x} dx$ | (26) $\int_0^1 \frac{1}{\sqrt{x}} dx$ | (27) $\int_0^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$ |
| (28) $\int_1^{\infty} \frac{1}{x^2} dx$ | (29) $\int_{-\infty}^{-1} \frac{2}{x^5} dx$ | (30) $\int_{-\infty}^{\infty} \frac{x}{(1+x^2)^2} dx$ |

2. Evaluate the following integrals.

$$(1) \int_{\frac{1}{2}}^{\infty} \frac{1}{x(\ln x)^{\frac{1}{5}}} dx \quad (2) \int_{-2}^2 \frac{1}{(x+1)^3} dx \quad (3) \int_0^2 \frac{1}{(x-1)^{\frac{2}{3}}} dx$$

$$(4) \int_0^4 \frac{1}{x^2 - 2x - 3} dx \quad (5) \int_{-2}^0 \frac{1}{(x+1)^{\frac{1}{3}}} dx \quad (6) \int_0^2 \frac{x}{1-x} dx$$

$$(7) \int_{-1}^2 \frac{1}{x^3} dx \quad (8) \int_0^{\pi} \frac{\sin x}{\sqrt[5]{\cos x}} dx \quad (9) \int_0^{\frac{\pi}{2}} \sec 2x dx$$

$$(10) \int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{\sqrt{1 - \tan x}} dx \quad (11) \int_{-1}^1 \frac{e^x}{\sqrt[5]{e^x - 1}} dx \quad (12) \int_1^4 \frac{1}{(x-2)^{\frac{2}{3}}} dx$$