WARM-UP

Multiple Choice Which of the following is an equation of the tangent line to $y = \sin x + \cos x$ at $x = \pi$?

(A) $y = -x + \pi - 1$ (B) $y = -x + \pi + 1$

(C) $y = -x - \pi + 1$ (D) $y = -x - \pi - 1$

(E) $y = x - \pi + 1$

Multiple Choice Find y'' if $y = x \sin x$.

(A) $-x \sin x$ (B) $x \cos x + \sin x$ (C) $-x \sin x + 2 \cos x$

(D) $x \sin x$ (E) $-\sin x + \cos x$

The Chain Rule

Objectives:

- Find the derivative of a composite function and a trigonometric function using the Chain Rule

Why do we need the Chain Rule?

Without the Chain Rule

$y = x^2 + 1$

$y = \sin x$

$y = 3x + 2$

$y = x + \tan x$

Why do we need the Chain Rule?

Without the Chain Rule	*With the Chain Rule*
$y = x^2 + 1$	$y = \sqrt{x^2 + 1}$
$y = \sin x$	$y = \sin 6x$
$y = 3x + 2$	$y = (3x + 2)^5$
$y = x + \tan x$	$y = x + \tan x^2$

Most functions can be written as a composition function. Every composition contains an inner part and an outer part.

$$f\left(g(x)\right)$$

f(x) ⇒ Outer Part

g(x) ⇒ Inner Part

The Chain Rule

$$\frac{d}{dx}\left[f\left(g(x)\right)\right] = f'\left(g(x)\right) \cdot g'(x)$$

It is helpful to think of the separate the function as having two parts - an inner part *g(x)* and an outer part *f(x)*.

$$p(x) = \left(5x^2 - 3\right)^8$$

Guided Practice:

Use the Quotient Rule to differentiate

$$y = \frac{2}{3x + 1}$$

Use the Chain Rule to differentiate

$$y = \frac{2}{3x + 1}$$

Differentiate

$$y = \csc(3x^2)$$

$$y = \sin(3x)\tan(2x)$$

Find the slope of the line tangent to the curve $y = \sin^5 x$ at the point where $x = \pi/3$

For the given function, determine all points for which *f′(x)* = 0 and those for which *f′(x)* does not exist.

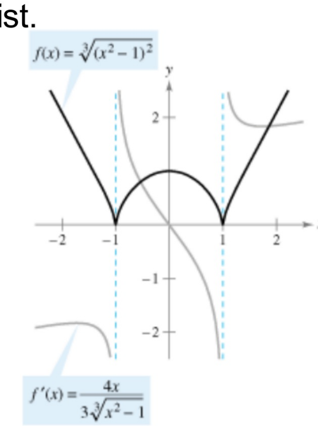
$$f(x)=\sqrt[3]{\left(x^2-1\right)^2}$$

For the given function, determine all points for which *f′(x)* = 0 and those for which *f′(x)* does no exist.

$$f(x)=\sqrt[3]{\left(x^2-1\right)^2}=\left(x^2-1\right)^{2/3}$$

$$f'(x)=\frac{2}{3}\left(x^2-1\right)^{-1/3}\left(2x\right)$$

$$f'(x)=\frac{4x}{3\sqrt[3]{x^2-1}}$$



The derivative of f is 0 at $x = 0$ and is undefined at $x = \pm 1$.

(1) $A(z) = (3z-5)^4$

(2) $g(z) = \dfrac{1}{\sqrt{36-z^2}}$

(3) $g(x) = \sqrt{4-3x^2}$

(4) $g(\theta) = \sec\left(\frac{1}{2}\theta\right)\tan\left(\frac{1}{2}\theta\right)$

(5) Find the tangent to the curve $y = 2\tan\left(\frac{\pi x}{4}\right)$ at $x = 1$.

(6) $y = \cos(1-2x)^2$

WARM-UP

Find the derivative of the following functions.

(1) $g(x) = x^2\left(\dfrac{2}{x} - \dfrac{1}{x+1}\right)$

(2) $y = \sqrt{x} + \frac{1}{4}\sin(2x)^2$

Now to put together everything we have learned to this point...

Differentiate

$$f(x)=x^2\sqrt{1-x^2}$$

Differentiate

$$f(x)=\frac{x}{\sqrt[3]{x^2+4}}$$

Differentiate

$$y=\left(\frac{3x-1}{x^2+3}\right)^2$$

Differentiate

$f(t) = \sin^3 4t$

E -

T -

A -

x	**1**	**3**	**6**
f(x)	4	0	6
f'(x)	5	7	4
g(x)	4	1	6
g'(x)	5	0.5	3

At x = 6, find the derivative of f(g(x)) =

x	**1**	**3**	**6**
f(x)	4	0	6
f'(x)	5	7	4
g(x)	4	1	6
g'(x)	5	0.5	3

At x = 1, find the derivative of f(2x + g(x)) =

Guided Practice

(1) $y = \frac{1}{2}x^2\sqrt{16 - x^2}$

(2) $h(t) = 2\cot^2(\pi t + 2)$

(3) $g(x) = \left(\frac{3x^2 - 2}{2x + 3}\right)^3$

(4) **Let $h(x) = f(g(x))$ and $s(x) = g(f(x))$**

(a) Find $h'(1)$.

(b) Find $s'(5)$.