Name:_________________________
AP Calculus
Unit 4: Mid-Unit Review

Section 1: Extreme Value Theorem.

Find the absolute extrema of *f* on the given interval using the Extreme Value Theorem. Verify the Extreme Value Theorem applies before evaluating (you do not need to justify that it applies).

(1) $f(x) = 2x^3 - 3x^2 - 12x + 1, [-2,3]$

(2) $f(x) = (x^2 - 1)^3, [-1,2]$

(3) $f(t) = \sqrt[3]{t}(8 - t), [0,8]$

(4) $f(x) = \sin x + \cos x\,, \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Section 2: Mean Value Theorem

(5) Determine if the function $f(x) = x\sqrt{6 - x}$ satisfies the hypothesis of Rolle's Theorem on the interval [0,6], and if it does, find all value *x* = *c* satisfying the conclusion of that theorem.

(6) Let *f* be a function defined on [-1,1] such that $f(-1) = f(1)$. Consider the following properties that *f* might have:

I. *f* is continuous on [-1,1], differentiable on (-1,1).
II. $f(x) = \cos^3 x$
III. $f(x) = |\sin(\pi x)|$

Which properties ensure that there exists a *c* in (-1,1) at which $f'(c) = 0$?

(A) I only (B) I and II only (C) I and III only (D) II and III only (E) I, II, and III

(7) Which of the following function below satisfy the hypothesis of the Mean Value Theroem?

I. $f(x) = \frac{1}{x+1}$ on [0,2]
II. $f(x) = x^{\frac{1}{3}}$ on [0,1]
III. $f(x) = |x|$ on [-1,1]

(A) I only (B) I and II only (C) I and III only (D) II and II only

(E) None of these satisfy the hypothesis of the Mean Value Theorem

Determine if the Mean Value Theorem can be applied to the following functions. If so, find the exact value(s) guaranteed by the theorem.

(8) $f(x) = \begin{cases} \arctan x\,, -1 \le x < 1 \\ \frac{x}{2},\ \ 1 \le x \le 3 \end{cases}$

(9) $f(x) = \ln(x - 1)\,, [2, e + 1]$

Section 3: First Derivative Test

(10) The derivative of a function *f* is given for all *x* be $f'(x) = (2x^2 + 4x - 16)\left(1 + g^2(x)\right)$ where g is some unspecified function. At which value(s) of *x* will *f* have a local maximum?

(A) *x* = -4 (B) *x* = 4 (C) *x* = -2 (D) *x* = 2 (E) *x* = -4, 2

Determine the local extrema of each of the following functions. Justify each solution.

(11) $f(x) = \cos^2(2x)$ on the interval $[0, \pi]$

(12) $f(x) = x + \frac{1}{x}$

Section 4: Second Derivative Test

(13) If $a < 0$, the graph of $y = ax^3 + 3x^2 + 4x + 5$ is concave up on which of the following intervals?

(A) $\left(-\infty, -\frac{1}{a}\right)$ (B) $\left(-\infty, \frac{1}{a}\right)$ (C) $\left(-\frac{1}{a}, \infty\right)$ (D) $\left(\frac{1}{a}, \infty\right)$ (E) $(-\infty, -1)$

x	0	2	4	6
$f''(x)$	2	6	0	-2

(14)
The polynomial function f has selected values on its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval (2,6).
(B) f is decreasing on the interval (2,6)
(C) The graph has a point of inflection at $x = 4$.
(D) f has a local maximum at $x = 4$.
(E) The graph of f changes concavity in the interval (2,6).

(15) Let f be a function with a second derivative given by $f''(x) = x^2(x+4)(x-7)$. What are the x-coordinates of the points of inflection of the graph of f?

(A) 0 only (B) -4 only (C) 0 and 7 only (D) -4 and 7 only (E) 0, -4, and 7

(16) The function f is continuous on the closed interval [3,5] and twice differentiable on the open interval (3,5). If $f'(4) = 2$ and $f''(x) < 0$ on the open interval (3,5), which of the following could be a table for f?

(A)

x	$f(x)$
3	2.5
4	5
5	6.5

(B)

x	$f(x)$
3	2.5
4	5
5	7

(C)

x	$f(x)$
3	3
4	5
5	6.5

(D)

x	$f(x)$
3	3
4	5
5	7

(E)

x	$f(x)$
3	3.5
4	5
5	7.5