WARM UP

1. Write the first four non zero terms and general term of the Taylor series for $f(x) = e^{2x}$ centered at c = 1.

2. Evaluate $\lim_{x \to \infty} e^{-x}\sqrt{x}$.

# Power Series - Part II
# Geometric Series

Objective:

- Create power series using Taylor's Rule, Long Division, and as a Geometric Series.

Write out the power series centered at c = 0 and find the radius and interval of convergence (using Taylor's Rule)

$$\frac{1}{1-x}$$

## Geometric Power Series

In this section and the next, you will study several techniques for finding a power series that represents a function. Consider the function

$$f(x) = \frac{1}{1-x}.$$

The form of $f$ closely resembles the sum of a geometric series

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}, \quad 0 < |r| < 1.$$

In other words, when $a = 1$ and $r = x$, a power series representation for $1/(1-x)$, centered at 0, is

$$\begin{aligned} \frac{1}{1-x} &= \sum_{n=0}^{\infty} ar^n \\ &= \sum_{n=0}^{\infty} x^n \\ &= 1 + x + x^2 + x^3 + \cdot \cdot \cdot, \quad |x| < 1. \end{aligned}$$

Find a power series centered at x = 0, then find the interval of convergence. Include the first four nonzero terms and the general term.

$$\frac{1}{1-2x}$$

Find a power series centered at x = 0, then find the interval of convergence. Include the first four nonzero terms and the general term.

$$\frac{x}{1+x}$$

Find a power series centered at x = 0, then find the interval of convergence. Include the first four nonzero terms and the general term.

$$\frac{1}{4+x}$$

6. Let $f$ be a function with derivatives of all orders and for which $f(2) = 7$. When $n$ is odd, the $n$th derivative of $f$ at $x = 2$ is 0. When $n$ is even and $n \geq 2$, the $n$th derivative of $f$ at $x = 2$ is given by $f^{(n)}(2) = \frac{(n-1)!}{3^n}$.

(a) Write the sixth-degree Taylor polynomial for $f$ about $x = 2$.

(b) In the Taylor series for $f$ about $x = 2$, what is the coefficient of $(x-2)^{2n}$ for $n \geq 1$?

(c) Find the interval of convergence of the Taylor series for $f$ about $x = 2$. Show the work that leads to your answer.