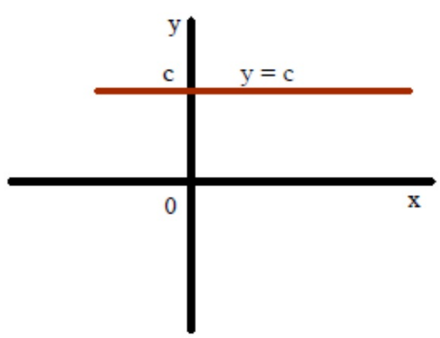
3.3 Rules for Differentiation

Objective:
Find the derivative of functions using the Power Rule.

Write equations of tangent and normal lines.

What's the slope of the line $y = c$?
So what would the slope of the tangent line be?



Thus, the derivative of any constant is zero.

$$\frac{d}{dx}(c) = 0$$

<u>The Power Rule for Derivatives</u>

$$\frac{d}{dx}\left(x^n\right) = n\,x^{n-1}$$

$f(x) = 3x^2$

$y = 6x$

$y = 5x^3 + 2x^2 - 6x + 5$

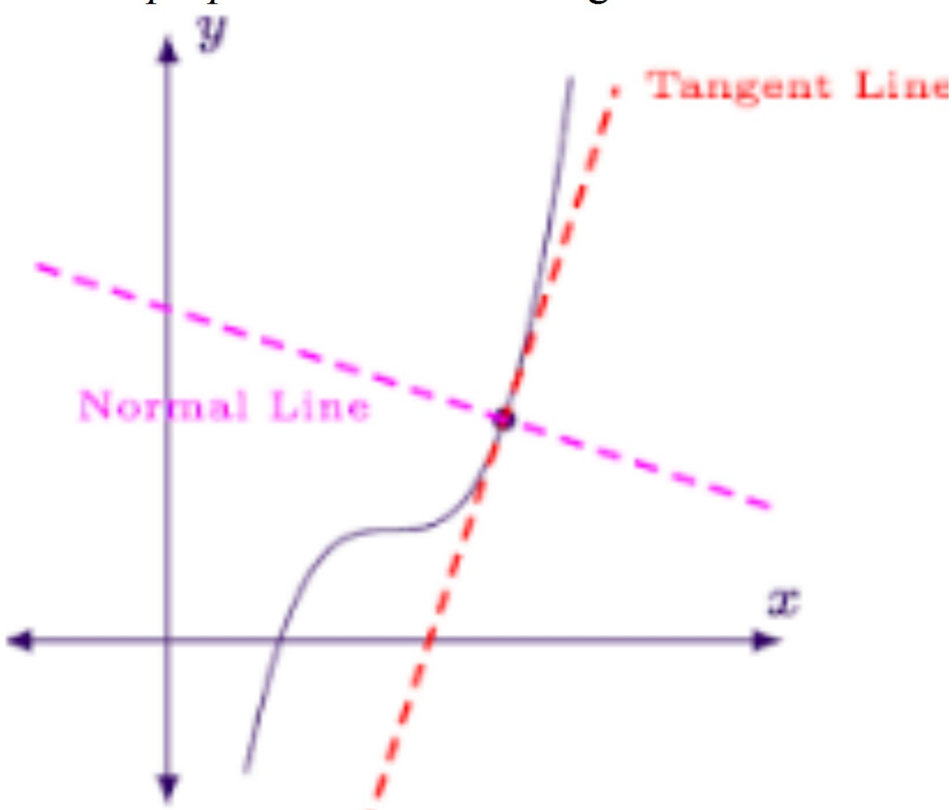
Hint: Sometimes its helpful to rewrite the function before using the power rule
(radicals and exponents in the denominator)

$$f(x) = \frac{1}{x^2}$$

$$f(x) = 2x^6 + 3\sqrt{x}$$

$$y = \frac{4x^2 - 3x + 2\sqrt{x}}{x}$$

The **normal line** to a curve at a particular point is the line through that point and *perpendicular* to the tangent.



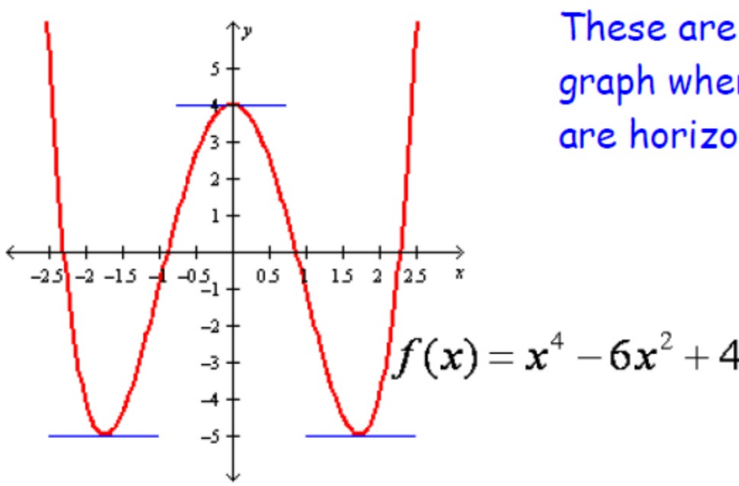
Find the equation of the line tangent to the curve
$y = 2x^3 - 2x^2 + 4x - 5$ at the point $x = 3$.

Find the equation of the line normal to the curve
$y = 2x^3 - 2x^2 + 4x - 5$ at the point $x = 3$.

EXAMPLE:

Find the points on the graph of $f(x)=x^4-6x^2+4$ where the tangent line is horizontal.

Plug these points back into the orginal equation and we get coordinates (x,y).



These are the points on the graph where the tangent lines are horizontal

A function is differentiable at *x* = *c*, if:

(1) The function is continuous at *x* = *c*

$$\lim_{x \to c} f(x) = f(c)$$

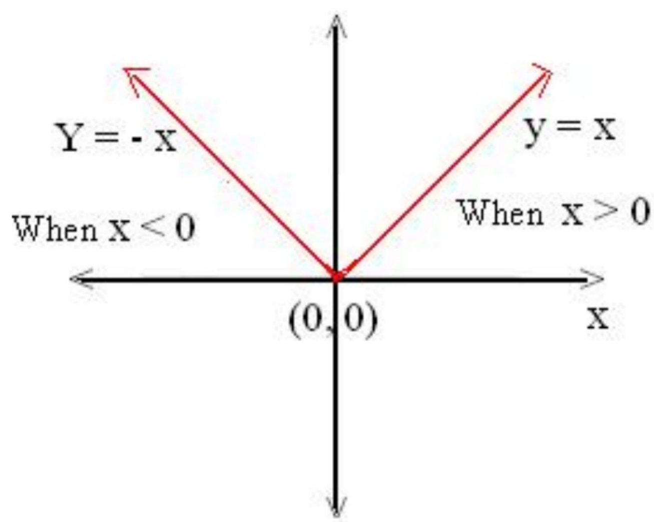
AND

(2) $\lim_{x \to c^-} f'(x) = \lim_{x \to c^+} f'(x)$

When is a function NOT differentiable?

1. a *corner* where the one-sided derivatives differ.

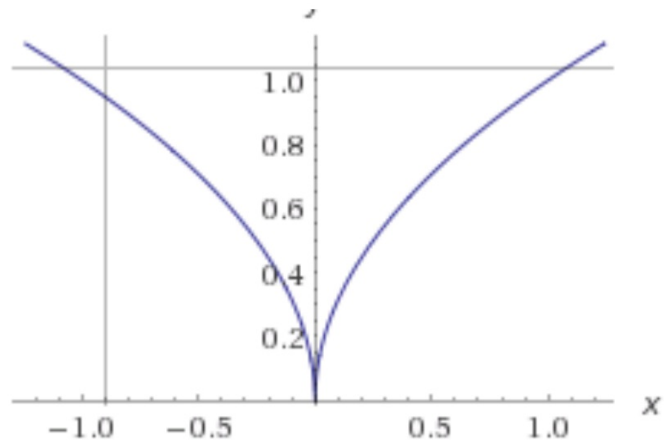
Ex: f(x) = |x|



When is a function NOT differentiable?

2. a *cusp*, an extreme case of a corner

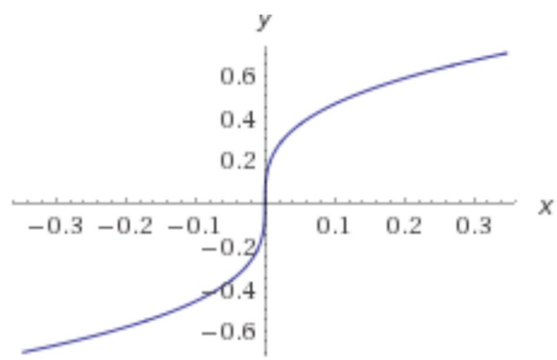
Ex: $f(x) = \sqrt{|x|}$



When is a function NOT differentiable?

3. a *vertical tangent line*, where the slope of the secant line approaches either ∞ or -∞ from both sides

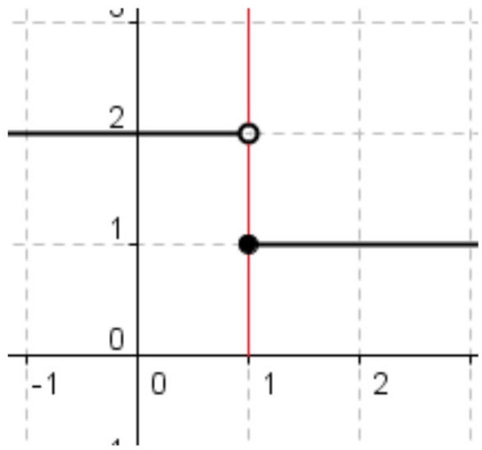
Ex: $f(x) = \sqrt[3]{x}$



When is a function NOT differentiable?

4. a *discontinuity*, either one or both sides of the one-sided derivatives is non-existent

Ex:



At $x = 4$, the function given by $h(x) = \begin{cases} x^2, & x \leq 4 \\ 4x, & x > 4 \end{cases}$ is

(A) Undefined

(B) Continuous but not differentiable

(C) Differentiable but not continuous

(D) Neither continuous nor differentiable

(E) Both continuous and differentiable

Key Concept

Differentiability implies continuity.

Continuity does not imply differentiability.

$$f(x) = \begin{cases} 3x - 2, x < -1 \\ -x - 6, x \geq -1 \end{cases}$$

1. Is the piecewise function, *f(x),* continuous at x = -1?

2. Is the piecewise function, *f(x)*, differentiable at x = -1?

$$f(x) = \begin{cases} cx + d & \text{for } x \leq 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$$

Let f be the function defined above, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

(A) –4 (B) –2 (C) 0 (D) 2 (E) 4