WARM UP

1. Find the extreme values of $g(x) = e^{-x}$ on the interval [-1,1].

2. What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point at which $x = 1/4$?

A. 2 B. 1/2 C. 0 D. -1/2 E. -2

3. $\frac{d}{dx}\left[\cos^2\left(x^3\right)\right] =$

(A) $6x^2\sin\left(x^3\right)\cos\left(x^3\right)$ (B) $6x^2\cos\left(x^3\right)$ (C) $\sin^2\left(x^3\right)$

(D) $-6x^2\sin\left(x^3\right)\cos\left(x^3\right)$ (E) $-2\sin\left(x^3\right)\cos\left(x^3\right)$

Mean Value Theorem and Rolle's Theorem

Objective:

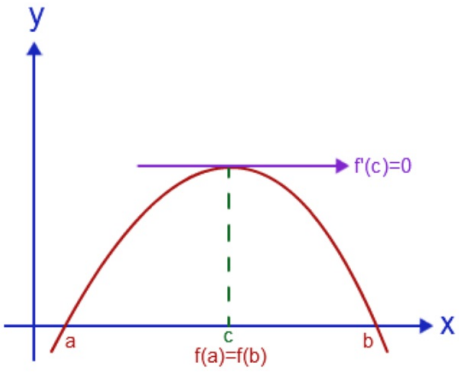
- Understand and apply Rolle's Theorem.
- Understand and apply Mean Value Theorem.

Rolle's Theorem:

Let f be a function that satisfies the following three hypothesis:

1. f is continuous on the closed interval $[a,b]$.
2. f is differentiable on the open interval (a,b).
3. $f(a)=f(b)$

Then there is a number $x=c$ in (a,b) such that $f'(c)=0$



Example 1: For $f(x) = x^4 - 2x^2$ on [-2,2], determine if Rolle's Thm applies. If so, find the value(s) of x=c guaranteed by the theorem.

Example 2: Determine if Rolle's Thm applies. If so, find the value(s) of x=c guaranteed by the theorem.

$$f(x) = x^{2/3} - 1, \quad [-8, 8]$$

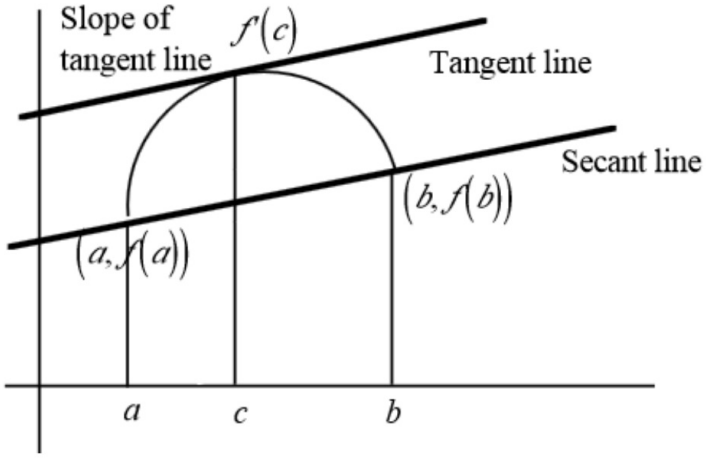
The Mean Value Theorem:

Let f be a function that satisfies the following two hypothesis:

1. f is continuous on the closed interval $[a,b]$.
2. f is differentiable on the open interval (a,b).

Then there is a number $x=c$ in (a,b) such that $f'(c)=\dfrac{f(b)-f(a)}{b-a}$

Or $f(b)-f(a)=f'(c)(b-a)$



Example 3: Determine if the MVT applies to $f(x) = x^3 - x - 1$ on the interval [-1,2]. If so, find the values guaranteed by the MVT.

Example 4: Determine if the MVT applies to f(x), if so, find the value(s) guaranteed by the theorem.

$$f(x) = x - 2 \sin x, \quad [-\pi, \pi]$$

Determine if the MVT applies, if so find the value(s) guaranteed by the theorem.

Example 5: $f(x) = \dfrac{x+5}{x-1}$ on [-3,5]

Example 6: Let f(x) be a function that is differentiable for all x. Suppose that f(0) = -3 and f '(x) ≤5 for all values of x. How large can f(2) be?

GUIDED PRACTICE

For #1-3, apply the MVT to f(x) on the interval.
Find all values of c which satisfy MVT.

1. f(x) = x^3 - 2x on [-1,1]

2. $f(x) = \sqrt{x-3}$ on $[3,7]$

3. $f(x) = \dfrac{x+2}{x}$ on $\left[\dfrac{1}{2},2\right]$

4. Find the extrema of:

$$f(x) = 3x^{2/3} - 2x + 1 \quad [-1,8]$$

3. Determine if the function $f(x) = x^3 - x - 1$ satisfies the hypothesis of the MVT on $[-1,2]$. If it does, find all possible values of c satisfying the conclusion of the MVT.