

Integration by Substitution

$$(1) \int \frac{x^2}{(1+x^3)^2} dx \quad u = 1+x^3$$
$$\frac{du}{3} = \frac{3x^2 dx}{3}$$

$$\frac{1}{3} \int u^{-2} du$$

$$\frac{1}{3} \left(\frac{u^{-1}}{-1} \right) + C$$

$$-\frac{1}{3(1+x^3)} + C \checkmark$$

$$(2) \int 10x(5x^2-4)^{\frac{1}{2}} dx \quad u = 5x^2-4$$
$$du = 10x dx$$

$$\int u^{\frac{1}{2}} du$$

$$\frac{2}{3} u^{\frac{3}{2}} + C = \frac{2}{3} (5x^2-4)^{\frac{3}{2}} + C \checkmark$$

$$(3) \int \frac{e^{\frac{1}{x}}}{x^2} dx = \int e^{\frac{1}{x}} \cdot \frac{1}{x^2} dx$$

$$u = \frac{1}{x} = x^{-1}$$
$$\frac{du}{-1} = \frac{-1x^{-2} dx}{-1}$$

$$-\int e^u du$$

$$-e^u + C$$

$$-e^{\frac{1}{x}} + C \checkmark$$

$$(4) \int \frac{\ln x}{x} dx \quad u = \ln x$$

$$du = \frac{1}{x} dx$$

$$\int u du = \frac{u^2}{2} + C = \frac{(\ln x)^2}{2} + C \checkmark$$

$$(5) \int \tan x dx = \int \frac{\sin x}{\cos x} dx \quad u = \cos x$$

$$\frac{du}{-1} = \frac{-\sin x dx}{-1}$$

$$-\int \frac{1}{u} du$$

$$-\ln|u| + C = -\ln|\cos x| + C \quad \text{or } \ln|\sec x| + C$$

~~$$(6) \int x^2 \cdot 3^{5x^3+1} dx \quad u = 5x^3 + 1$$

$$\frac{du}{15} = \frac{15x^2 dx}{15}$$~~

~~$$\frac{1}{15} \int 3^u du$$~~

$$(7) \int \frac{\arcsin x}{\sqrt{1-x^2}} dx$$

$$u = \arcsin x$$

$$du = \frac{1}{\sqrt{1-x^2}} dx$$

$$\int u^{1/2} du$$

$$\frac{2}{3} u^{3/2} + C = \frac{2}{3} (\arcsin x)^{3/2} + C$$

(8)

$$\int \frac{x}{\sqrt{1-x^4}} dx = \int \frac{x}{\sqrt{1-(x^2)^2}} dx \quad u = x^2$$

$$\frac{du}{2} = \frac{2x dx}{2}$$

$$\frac{1}{2} \int \frac{1}{\sqrt{1-u^2}} du = \frac{1}{2} \arcsin u + C$$

$$= \frac{1}{2} \arcsin(x^2) + C$$

$$\textcircled{9} \int \frac{3x+4}{\sqrt{3x^2+8x+3}} dx \quad u = 3x^2+8x+3$$

$$du = (6x+8)dx$$

$$\frac{du}{2} = \frac{2(3x+4)dx}{2}$$

$$\frac{1}{2} \int u^{-1/2} du$$

$$\frac{1}{2} (2) u^{1/2} + C$$

$$(3x^2+8x+3)^{1/2} + C$$

$$\textcircled{11} \int \sqrt{1+2x} dx \quad u = 1+2x$$

$$\frac{du}{2} = \frac{2dx}{2}$$

$$\frac{1}{2} \int u^{1/2} du$$

$$\frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C = \frac{1}{3} (1+2x)^{3/2} + C$$

$$\textcircled{13} \int x \sqrt{2x^2+7} dx \quad u = 2x^2+7$$

$$\frac{du}{4} = \frac{4x dx}{4}$$

$$\frac{1}{4} \int u^{1/2} du$$

$$\frac{1}{4} \cdot \frac{2}{3} u^{3/2} + C = \frac{1}{6} (2x^2+7)^{3/2} + C$$

$$\textcircled{14} \int (2x^{3/2}+1)x^{1/2} dx \quad u = 2x^{3/2}+1$$

$$\frac{1}{3} \int u du$$

$$\frac{1}{3} \frac{u^2}{2} + C = \frac{1}{6} (2x^{3/2}+1)^2 + C$$

$$(15) \int \frac{\sin 3x}{\sqrt{5 + \cos 3x}} dx$$

$$u = 5 + \cos 3x$$
$$\frac{du}{-3} = \frac{-3 \sin 3x dx}{-3}$$

$$\frac{1}{3} \int u^{-1/2} du$$

$$\frac{1}{3} \cdot 2 u^{1/2} + C = \frac{2}{3} (5 + \cos 3x)^{1/2} + C$$

$$(16) \int \frac{e^x}{4 - e^x} dx$$

$$u = 4 - e^x$$
$$-du = -e^x dx$$

$$\int \frac{1}{u} du$$

$$-\ln|u| + C = -\ln|4 - e^x| + C$$

$$(17) \int \frac{2x-3}{x^2-3x+5} dx$$

$$u = x^2 - 3x + 5$$
$$du = (2x - 3) dx$$

$$\int \frac{1}{u} du = \ln|u| + C = \ln|x^2 - 3x + 5| + C$$

$$(18) \int x \sqrt{x^2 + 9} dx$$

$$u = x^2 + 9$$
$$\frac{du}{2} = \frac{2x dx}{2}$$

$$\frac{1}{2} \int u^{1/2} du$$

$$\frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C = \frac{1}{3} (x^2 + 9)^{3/2} + C$$

$$\textcircled{19} \int x e^{x^2} dx \quad u = x^2$$

$$\frac{du}{2} = \frac{2x dx}{2}$$

$$\frac{1}{2} \int e^u du$$

$$\frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$

$$\textcircled{20} \int (\sin x)^2 \cos x dx \quad u = \sin x$$

$$du = \cos x dx$$

$$\int u^2 du$$

$$\frac{u^3}{3} + C = \frac{(\sin x)^3}{3} + C$$

$$\textcircled{21} \int (x+5)^6 dx \quad u = x+5$$

$$du = 1 dx$$

$$\int u^6 du$$

$$\frac{1}{7} u^7 + C = \frac{1}{7} (x+5)^7 + C$$

$$\textcircled{22} \int (3x-2)^9 dx \quad u = 3x-2$$

$$\frac{1}{3} du = \frac{3 dx}{3}$$

$$\frac{1}{3} \int u^9 du$$

$$\frac{1}{3} \frac{u^{10}}{10} + C = \frac{1}{30} (3x-2)^{10} + C$$

$$(23) \int \frac{3x^2}{\sqrt{x^3+7}} dx \quad u = x^3+7$$

$$du = 3x^2 dx$$

$$\int u^{-1/2} du$$

$$2u^{1/2} + C = 2(x^3+7)^{1/2} + C$$

$$(24) \int \frac{(\ln x)^5}{x} dx \quad u = \ln x$$

$$du = \frac{1}{x} dx$$

$$\int u^5 du$$

$$\frac{u^6}{6} + C = \frac{(\ln|x|)^6}{6} + C$$

$$(25) \int \frac{1}{x \ln x} dx \quad u = \ln x$$

$$du = \frac{1}{x} dx$$

(26) at end ;

$$\int \frac{1}{u} du = \ln u + C = \ln|\ln|x|| + C$$

$$(27) \int (2x+5)(x^2+5x)^7 dx \quad u = x^2+5x$$

$$du = (2x+5) dx$$

$$\int u^7 du$$

$$\frac{u^8}{8} + C = \frac{(x^2+5x)^8}{8} + C$$

$$(28) \int (3-4x)^8 dx \quad u = 3-4x$$

$$\frac{du}{-4} = \frac{-4 dx}{-4}$$

$$-\frac{1}{4} \int u^8 du$$

$$-\frac{1}{4} \cdot \frac{1}{9} u^9 + C = -\frac{1}{36} (3-4x)^9 + C$$