WARM UP:

Given the following sequences, write an expression for the nth term.

1. 3,8,13,18,...

2. 5, -15, 45, -135, ...

3. 2/1, 3/3, 4/5, 5/7, 6/9, ...

4. List the following funtions in order from growing the slowest to the fastest as they approach infinity:

Polynomial, ln, x^x, exponential, factorial, constant

Intro to Series

Objective:

- Review sequences and series (notation)
- Determine if a sequence converges or diverges.

SEQUENCES

A sequence is simply a list of numbers generated by a rule.

$$a_1, a_2, \ldots\, a_n, \ldots\ldots$$

a_n is called the *nth* term.

Ways to write a sequence:

- list out a numer of terms in the sequence, separated by comas
- an explicit function, such as $a_n =$
- rule of the sequence set off in braces $\{a_n\}$

If $a_n = \left\{ \dfrac{4n}{3+2n} \right\}$ list out the first five terms, then estimate $\lim\limits_{n\to\infty} a_n$

Let $\{a_n\}$ be a sequence of real numbers.

Possibilities:

1) If $\lim\limits_{n\to\infty} a_n = \infty$, then $\{a_n\}$ diverges to infinity
2) If $\lim\limits_{n\to\infty} a_n = -\infty$, then $\{a_n\}$ diverges to negative infinity
3) If $\lim\limits_{n\to\infty} a_n = c$, an finite real number, then $\{a_n\}$ converges to c
4) If $\lim\limits_{n\to\infty} a_n$ oscillates between two fixed numbers, then $\{a_n\}$ diverges by oscillation

$n!$ is read as "n factorial." It is defined recursively as $n! = n(n-1)!$ or as

$$n! = n(n-1)! = n(n-1)(n-2)(n-3)\cdots 3\cdot 2\cdot 1$$

Determine whether the following sequences converge or diverge.

(a) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots, \frac{n}{n+1}, \ldots$

(b) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots, \frac{1}{2^n}, \ldots$

(c) $a_n = 3 + (-1)^n$

(d) $a_n = \frac{n}{1-2n}$

(e) $a_n = \frac{\ln n}{n}$

(f) $a_n = \frac{n!}{(n+2)!}$

(g) $a_n = \dfrac{2n!}{(n-1)!}$

(h) $a_n = \dfrac{n+(-1)^n}{n}$

(i) $a_n = \dfrac{(-1)^n (n-1)}{n}$

(j) $a_n = \dfrac{2^n}{(n+1)!}$

(k) $a_n = \left(1+\dfrac{1}{n}\right)^n$

(l) $\left\{\dfrac{(2n)!}{n^n}\right\}$

Series

A series is the sum of the terms in a sequence.

$a_1 + a_2 + ...+ a_n + ...$

There are finite and infinite series.

Partial Sums

$$s_1 = a_1$$
$$s_2 = a_1 + a_2$$
$$s_3 = a_1 + a_2 + a_3$$
$$s_4 = a_1 + a_2 + a_3 + a_4$$
$$\vdots$$
$$s_n = a_1 + a_2 + a_3 + a_4 + \cdots + a_n = \sum_{i=1}^{n} a_i$$

The S_n are called partial sums. Notice that the partial sum is a sequence.

Summation notation: $$\sum_{k=1}^{n} a_k$$

read as "the sum of a_k from k = 1 to n"

k = the index of summation

Generate several of partial sums for the series $a_n = 1/n$ and $a_n = 1/n^2$, and then determine if the infinite series would converge or diverge.