WARM UP

1. $\lim_{x\to-\infty} \dfrac{4x^5+2x^2-3x+1}{\sqrt{9x^{10}+11x^9+12x^2+13x+14}}$ (D) $-\dfrac{4}{3}$ (E) $\dfrac{4}{9}$

(A) $-\infty$ (B) ∞ (C) $\dfrac{4}{3}$

2. On the interval [-π,π], find where f '(x) = 0 for the function f(x) = cosx - cos^{2}x

3. $\lim_{h\to 0} \dfrac{3\sec(\pi+h)-3\sec\pi}{h}$ (D) π (E) DNE

(A) -1 (B) 0 (C) -3

Extreme Values of a Function (Extrema)

Objective:

- Identify critical numbers and find extrema on an interval.

Definition: (Absolute/Global) Extrema/Extreme y-values

If f is a function on an interval I, then $y = f(c)$ is the

I. (Absolute/Global) **Maximum** on I, IFF $f(c) \geq f(x)$ for all x in I.

II. (Absolute/Global) **Minimum** on I, IFF $f(c) \leq f(x)$ for all x in I.

The Extreme Value Theorem (EVT)

If f is continuous on a closed interval $[a,b]$, then f has both a maximum value and a minimum value on the interval $[a,b]$.

Definition: Relative/Local Extrema

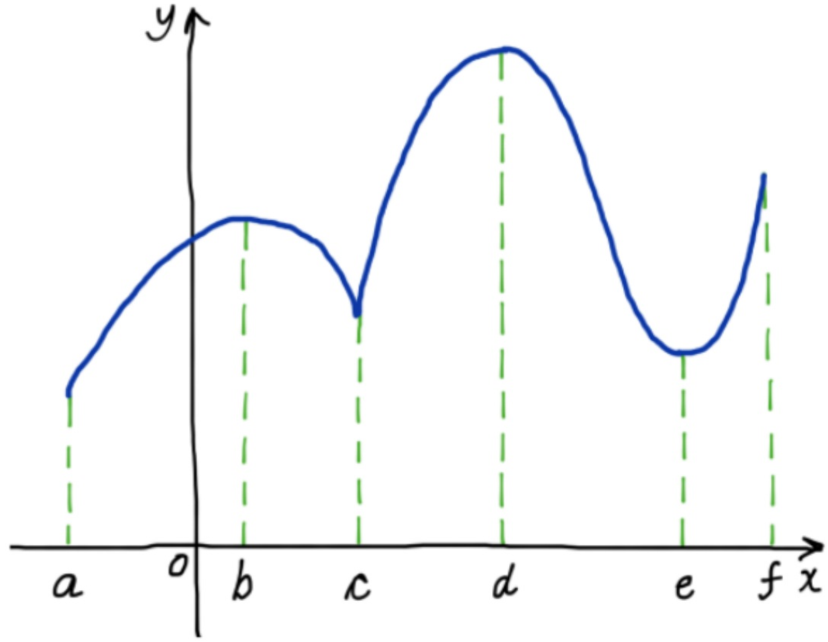
If there exists an **open** interval containing $x = c$, then

I. If immediately to the left of $x = c$ and immediately to the right of $x = c$ there are no y-values **greater** than $f(c)$, then $f(c)$ is a **relative/local maximum** of f.

II. If immediately to the left of $x = c$ and immediately to the right of $x = c$ there are no y-values **smaller** than $f(c)$, then $f(c)$ is a **relative/local minimum** of f.

Example 1:

The graph of $g(x)$ is given below. Determine the extrema of $g(x)$ on the interval $x \in [a, f]$.



Definition: Critical Value

A **critical value** of a function f is a value $x = c$ in the domain of f such that either

$$f'(c) = 0 \text{ or } f'(c) = DNE$$

If $x = c$ is a critical value, then $(c, f(c))$ is a critical point.

Example 2: Determine all the critical points:

$$h(x) = \sin^2 x + \cos x$$

$$0 < x < 2\pi$$

Example 3: Determine all the critical points:

$$g(t) = \sqrt[3]{t^2}(2t - 1)$$

Closed Interval Method for finding Extrema

To find absolute extrema of a continuous function $f(x)$ on a closed interval $[a,b]$.

1. Identify the endpoints.
2. Identify any critical values in (a,b)—**verify they are in the <u>domain</u> & the <u>specified interval</u>!!**
3. Find the function values, $f(x)$, at both endpoints and at all critical values in (a,b).
4. The largest value will be the max. The smallest value will be the min.
5. Answer the question asked in a complete sentence.

Example 4: Find the extrema of $f(x) = 3x^4 - 12x^3$ on the interval [-1,2]

Example 5: Find the extrema of f(x) = 2x - 3x$^{(2/3)}$ on the interval [-8,1]

Example 6:

. Find all the critical values of f when $f(x)=x^{4/5}(x-5)^2$. Be sure to factor out least powers after differentiating.

(A) 0, $\frac{5}{7}$ (B) $\frac{10}{7}$, 5 (C) $\frac{5}{7}$, 5 (D) 0, $\frac{5}{7}$, 5 (E) 0, $\frac{10}{7}$ (F) 0, $\frac{10}{7}$, 5

Example 7:

Let *f* be the function defined by $f(x)=\sin x-\cos^2 x$ on $[0,2\pi]$.

(iii) Determine the absolute maximum value of $f(x)$ on $[0,2\pi]$.

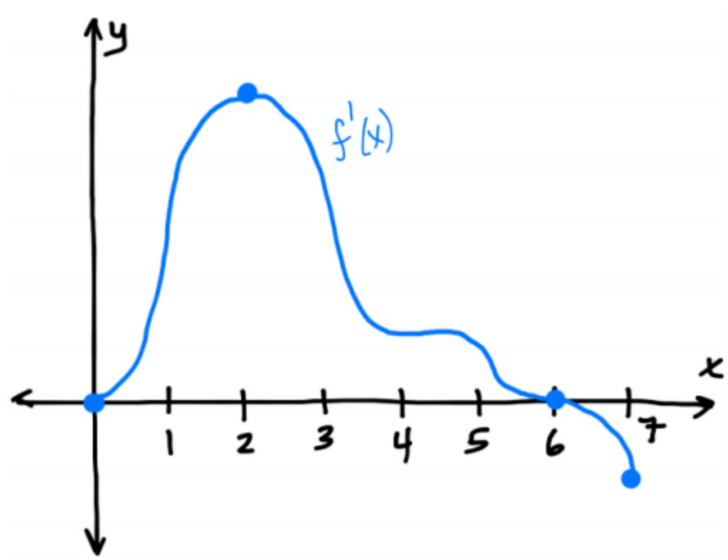
(A) -1

(B) $-\frac{5}{4}$

(C) $\frac{5}{4}$

(D) 1

Example 8:



The graph above is the graph of $f'(x)$, the derivative of some function $f(x)$. Use the graph above to determine the *x*-value at which the function $f(x)$ achieves its absolute maximum.

(A) 0 (B) 2 (C) 4 (D) 6 (E) 7