

Warm-up

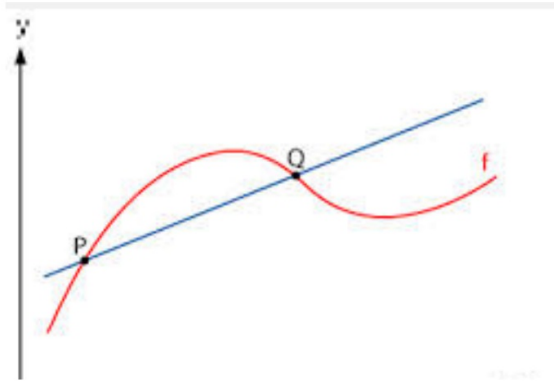
- (1) Find the slope between $(-2,5)$ and $(-7,12)$.**
- (2) Write the equation of a line in point-slope form that goes through the coordinate in #1.**
- (3) Write the equation in slope-intercept form that is perpendicular to the points in #1 and goes through the coordinate $(10,-4)$.**

Definition of a Derivative

Objective:

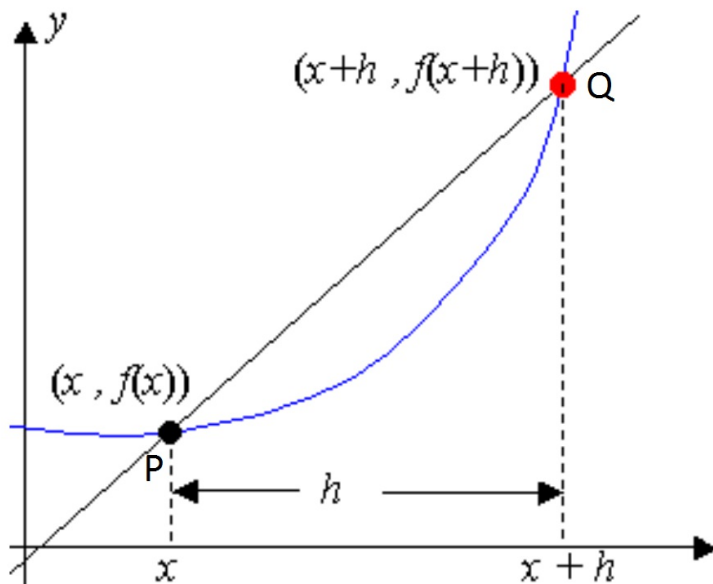
- Find the derivative of a function**
- Determine the qualifications for differentiability**

A line through two points on a curve is called a *secant to the curve* or *secant line*.

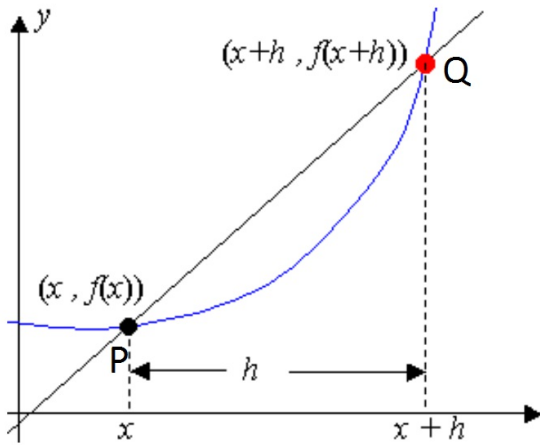


The average rate of change is also the slope of the secant line through two points.

Find the slope between P and Q.



What would the slope be just a point P?
(a line tangent to the curve at P)



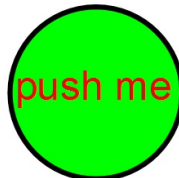
If we move Q closer and closer to P by making h become smaller and smaller, we get....

2.6 Slope of a Tangent Line
 The formula for the slope of the tangent line to $y = f(x)$ at the point $(a, f(a))$ is

$$m = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$m = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

We can generalize this formula for all possible values of x!



Definition of Derivative

f prime

the derivative of f

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

y_2

y_1

$x_2 - x_1$

If $f'(x)$ exists, we say that f has a derivative or is **differentiable**.

A function is differentiable at $x = c$, if:

(1) The function is continuous at $x = c$

$$\lim_{x \rightarrow c} f(x) = f(c)$$

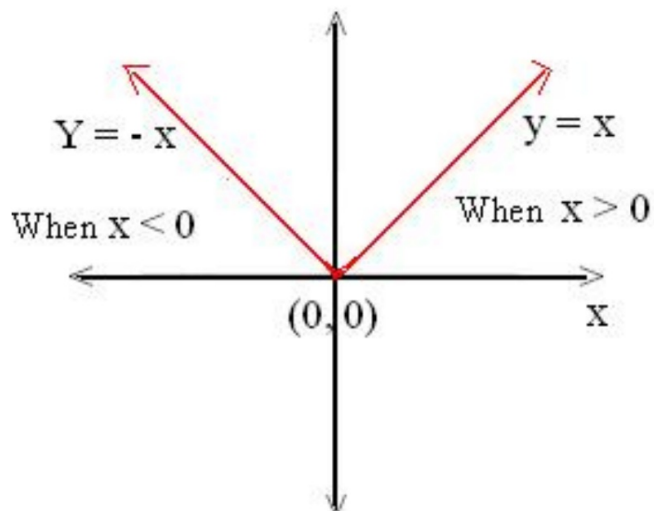
AND

$$(2) \lim_{x \rightarrow c^-} f'(x) = \lim_{x \rightarrow c^+} f'(x)$$

When is a function NOT differentiable?

1. a *corner* where the one-sided derivatives differ.

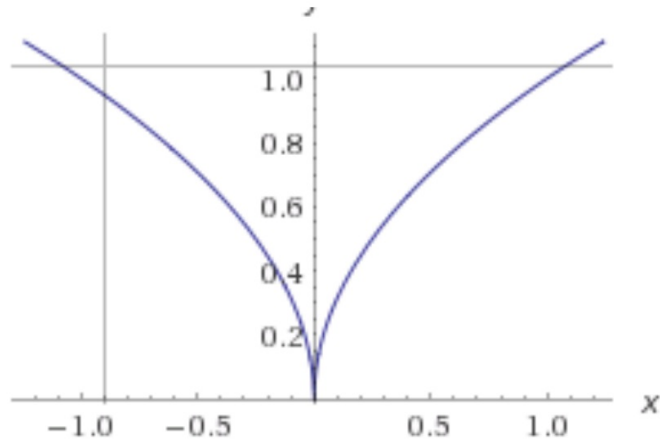
Ex: $f(x) = |x|$



When is a function NOT differentiable?

2. a *cusp*, an extreme case of a corner

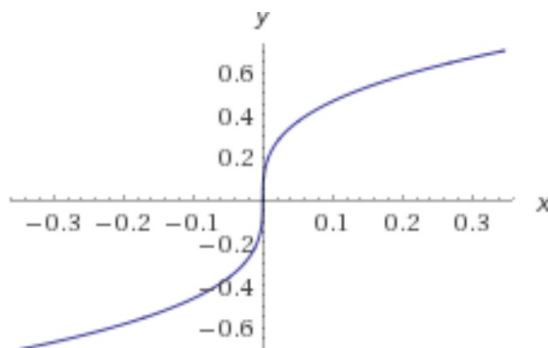
Ex: $f(x) = \sqrt{|x|}$



When is a function NOT differentiable?

3. a *vertical tangent line*, where the slope of the secant line approaches either ∞ or $-\infty$ from both sides

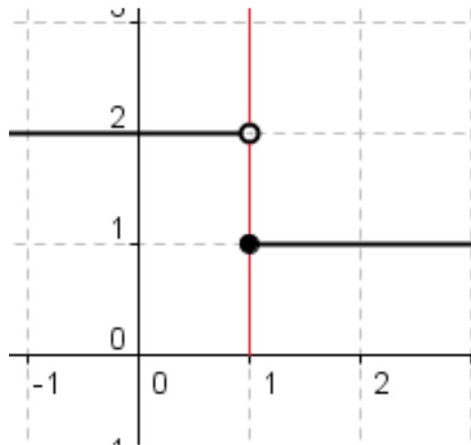
Ex: $f(x) = \sqrt[3]{x}$



When is a function NOT differentiable?

4. a *discontinuity*, either one or both sides of the one-sided derivatives is non-existent

Ex:



Notation

symbol

you say:

it means:

y'

"y prime"

nice and brief
doesn't always
mean independent
variable

$\frac{dy}{dx}$

"dy dx" or
"derivative of y
with respect to x"

names both variables
and used d for derivative

$\frac{d}{dx} f(x)$

"d dx of f at x"

emphasizes the idea that
differentiation is an
operation performed on f

$$f(x) = 3x - 2$$

$$f(x) = x^2 - x$$

$$f(x) = x^3 + 6$$

Find the slope of the tangent line of $f(x) = x^2 + 2x$ when $x = 3$.

$$f(x) = \frac{2}{x - 3}$$

$$f(x) = \sqrt{x - 1}$$